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# Investment and current utility change in dynamically inefficient economies

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## Abstract

An extensive literature shows the importance of investment policy for sustainability of resource-based economies by examining the role of investment in current utility change (CUC) for a competitive optimizing economy. This paper extends some of these results by analysing the dependence of CUC on genuine investment (GI), expressed in marginal resource productivity, under dynamic inefficiency. The inefficiency arises when a social planner, due to imperfection in knowledge or in institutions, does not take into account deviations of real economy from a theoretical model. These deviations or distortions, connected with the resource extraction, can influence utility, production, the balance equation, and the dynamics of the reserve. The analysis of this natural discrepancy between theory and real life implies that: first, institutional and resource policies in inefficient economies may be more important for CUC than investment policy; and secondly, under uncertainties in production possibilities and in damages from economic activities, sustainability requires a more cautious resource policy than is advised by a theory. The paper also suggests that the indicators GI, expressed in account-

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ing prices and in marginal resource productivity, can complement each other in sustainability evaluation.

*Key words:*

nonrenewable resource; dynamic inefficiency; genuine investment; resource policy; sustainable development

JEL : O13; O47; Q32; Q38

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## 1. Introduction<sup>1</sup>

Sustainability of real economies is always evaluated under uncertainties in future production possibilities and in various distortions such as institutional imperfections, economic wastefulness, damages from the processes of production and consumption to utility and to production itself. These uncertainties lead to the errors in the decisions of a social planner and cause deviations from an efficient and optimal path of economic development.

The literature on sustainability evaluation of resource-based economies offers an indicator of sustainable development, called genuine saving or genuine investment (GI), which is equal to increase in man-made capital minus resource depletion. This indicator was developed in the studies of the change in the current (present) value of consumption or utility at a specific moment of time in a dynamically efficient optimizing economy. Straightforward applications of these results to real-world situations can form the impression that, for sustainability, it is enough to invest in a proper way into man-made and human capital regardless of the pattern of extraction.

The current paper extends some of these studies by assuming that a real economy deviates from theoretical paths, but a planner, due to imperfection in knowledge or in institutions, uses the policies developed for an undistorted model. The paper provides the examples of distortions, which show that this natural discrepancy between a model and real life results in dynamically inefficient paths and, in some cases, unsustainability of the economy.

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<sup>1</sup>This study is a development of the author's paper "Investment and resource policy under a modified Hotelling rule."

The idea of the indicator GI was offered in Hartwick (1977) as the “invest resource rent” rule for the problem formulated in Solow (1974) for the Dasgupta-Heal-Solow-Stiglitz (DHSS)<sup>2</sup> model of a perfectly competitive resource-based economy satisfying the standard Hotelling rule (HR) as a necessary condition of dynamic efficiency. For this model, zero GI with resource depletion measured in market prices leads to constant per capita consumption over time. Dixit et al. (1980) extended the Hartwick rule by showing for a more general production function that GI that is constant over time in present prices is a necessary and sufficient condition for a constant path of utility.<sup>3</sup> Dasgupta and Heal (1979, pp. 303-306), Hamilton and Hartwick (2005), and Hamilton et al. (2006) analyzed the link between GI in current prices and current change in per capita consumption. Hamilton and Withagen (2007) derived the result of Dixit et al. (1980) (including also the result of Hamilton and Hartwick (2005)) in a more general setting (with internalized externalities), showing that instantaneous utility increases if and only if GI decreases in present prices. All these results were obtained for a competitive dynamically efficient economy.

Besides theoretical studies, variants of the indicator GI were used for practical evaluation of sustainability. For example, Pearce and Atkinson (1993) offered a simple indicator of weak sustainability<sup>4</sup> based on the assertion that

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<sup>2</sup>This model with the Cobb-Douglas production function, which includes a nonrenewable resource as a production factor, was developed in the works of Dasgupta and Heal (1974), Solow (1974), and Stiglitz (1974).

<sup>3</sup>Constant investment in present prices means that investment in current prices is growing with the rate of discount.

<sup>4</sup>Weak sustainability of growth (development) is defined by Pezzey (1992) as nonde-

“an economy is sustainable if it saves more than the *combined* depreciation on the two forms of capital” (man-made and natural). A variant of this indicator, modified for open economies, has been developed in Proops et al. (1999). These indicators were used in both papers to classify a number of countries into sustainable and unsustainable. Hamilton and Clemens (1999) developed a theory of genuine saving by adding the investment in human capital to traditional net savings and subtracting the value of resource depletion and environmental damage. The value of genuine saving was offered as an indicator of sustainability, and this indicator was used for comparing sustainability of a wide range of developing countries. Hence, as Hamilton and Hartwick (2005, p. 615) noted, “the magnitude of ‘net investment’ or ‘genuine savings’ has become a central focus in the measurement of the sustainability of an economy.”

Proposition 1 (Section 2) of this paper extends Proposition 1 of Hamilton and Hartwick (2005), providing the link between GI and current utility change (CUC) in a dynamically inefficient economy. The result shows that: 1) CUC may be determined only by the influence of inefficiency when this influence is not close to zero; 2) inefficiency asymmetrically affects the ability of investment to influence the sign of CUC, and this asymmetry is mutually inverted for large resource-poor and small resource-rich economies; 3) resource-based economies can be classified by the importance of investment or resource policies or both for CUC.

The examples of distortions (Section 3) include: 1) a resource-augmenting

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creasing per capita consumption (utility).

technical change (Takayama 1980) that distorts the optimal dynamics of the resource stock and leads to a sustainable but dynamically inefficient economy when a planner does not take it into account; 2) irreversible damages to utility and production resulting from the resource use (for example, Stollery 1998) that lead to inefficient and unsustainable outcomes when the planner ignores these effects; and 3) insecure property rights (Arrow et al. 2003) that also cause inefficient use of the resource and unsustainability of the economy, unless corrected by institutional transformations and resource policies.

The results of this study illustrate that, for sustainability, it is preferable to underestimate future production possibilities and overestimate damages since this policy of extra caution can reduce irreversible losses. Of course, this policy may lead to dynamic inefficiency caused by an overly conservative resource policy, but with updates in knowledge, the policy can be corrected, and the economy can be asymptotically efficient.

This paper also shows that an indicator linked to CUC may be, by construction, not sensitive to the changes in the ability of an economy to maintain non-declining utility during a long period of time (Section 4). According to Arrow et al. (2003), the accounting price<sup>5</sup> of a natural resource can be considerably higher for imperfect economies than the market price, implying that the investment of the market resource rent and even the entire market-valued output into man-made capital can be not enough to compensate for damages caused by the resource extraction. In other words, GI in accounting prices can be negative despite any effort in saving, suggesting that for

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<sup>5</sup>The accounting price of the resource stock shows the change in the intergenerational welfare when the resource stock is changed by one unit.

some inefficient economies, institutional and resource policies are prerequisites of sustainability. On the other hand, GI in accounting prices, also by construction, may be not sensitive to CUC, which also may lead to violations of sustainability. Therefore, different forms of GI can complement each other in sustainability evaluation.

## 2. Investment and growth under dynamic inefficiency

In order to define a distorted economy, it is instructive to introduce first a “perfect” or undistorted optimizing economy. Following Hamilton and Hartwick (2005, p. 618), assume that the economy is closed, time  $t$  is continuous, consumption is aggregated into a single good  $C$ , labor is fixed, so that output  $Q(t) = F(K, R)$  depends on man-made capital  $K(t)$  and the resource flow  $R(t) = -\dot{S}(t)$ , where  $S(t)$  is the current resource stock ( $\dot{S} := dS/dt$ ). The technology is stationary ( $F$  does not depend explicitly on  $t$ ).

A number of studies, which results were used for practical evaluation of sustainability, assume that the economy satisfies the following:

- $F(K, R)$  is a regular production function that (a) denotes the maximum output for the given  $K$  and  $R$ , and (b) satisfies the Inada conditions, in particular  $F_R > 0$  (*resource productiveness*), where  $F_R := \partial F / \partial R$ ;
- output  $Q$  equals  $F(K, R)$  (*static efficiency*);<sup>6</sup>
- the balance equation holds:  $C + \dot{K} = F(K, R) - \delta K$ , where  $\dot{K}$  is investment and  $\delta K$  with  $\delta = \text{const}$  is capital decay (*non-wastefulness*);

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<sup>6</sup>Conventionally, efficiency is defined via the Pareto-optimality. Some studies, e.g. Hurwicz (1960), called this notion non-wastefulness.



- the standard HR  $\dot{F}_R = rF_R$ <sup>7</sup> holds as a necessary condition of *dynamic efficiency*;
- the economy (a planner) maximizes a (social) welfare function by choosing the paths of  $\dot{K}$  and  $R$  (*optimality*).

In the real world, however, the resource use can be

- non-productive ( $F_R = 0$ ) or counter-productive ( $F_R < 0$ );<sup>8</sup>
- productive, but static-inefficient ( $Q < F(K, R)$ );
- productive, static-efficient, but wasteful: ( $C + \dot{K} < F(K, R) - \delta K$ );
- productive, static-efficient, non-wasteful, but dynamically inefficient;
- productive, non-wasteful, efficient, but not optimal.

This paper assumes that there is a vector  $\mathbf{D}(t) = (D_1(t), D_2(t), D_3(t), D_4(t))$ , called *distortion*, where  $D_i$  are the distortions in

$$\text{production:} \quad F = F(K, R, D_1), \quad (1)$$

$$\text{social utility:} \quad U = U(C, D_2), \quad (2)$$

$$\text{the balance equation:} \quad \dot{K} = F(K, R, D_1) - C - \delta K - D_3, \quad (3)$$

$$\text{the dynamics of the stock:} \quad \dot{S} = -R + D_4. \quad (4)$$

The distortions may include imperfections, externalities, and any effects (including favorable for sustainability) that cause violation of the standard HR.

To illustrate the claims of this study, it is enough to accept the following

**Assumptions:** 1)  $\mathbf{D}$  depends only on the extracted amount  $S_0 - S(t)$  or

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<sup>7</sup>Here,  $r(t) := F_K(t) - \delta$  is the market interest rate.

<sup>8</sup>The resource use is counter-productive when the decline in the resource stock results in the decline of output, e.g., as a result of a wildfire or oil spill.

2)  $\mathbf{D}$  depends on some parameters that do not depend on time explicitly.<sup>9</sup>

Hence,  $\mathbf{D}$  do not depend here on  $R$ ,<sup>10</sup>  $K$ , and  $C$ . For example,  $D_1$  and  $D_2$  may result from irreversible damages caused by economic activities (stock externalities, for example, due to climate change);  $D_3$  may stand for the growing cost of extraction (best-quality stock extracted first) or for static inefficiency and (or) for wastefulness of the economy ( $D_3 > 0$ );  $D_4$  may be the productivity of the stock-augmenting investment, which is, first, growing with the extraction due to learning-by-doing and eventually declining due to the scarcity of the resource.

Let  $D_5$  be a deviation of the ratio  $\dot{F}_R/F_R$  from a dynamically efficient path. Then the following result holds.

**Lemma 1.** *In economy (1)-(4),*

$$\dot{F}_R = [v(t) + \tau(t)] F_R, \quad (5)$$

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<sup>9</sup> $\mathbf{D}$  may depend, e.g., on the parameters that express institutional imperfections.

<sup>10</sup>In fact,  $\mathbf{D}$  may depend on  $R$ , e.g., when damage includes the opportunity cost (Gaudet et al., 2006), or when damage is partly reversible. Then formula (6) below is more complicated, which, however, does not alter the conclusions of the paper.  $\mathbf{D}$  can also depend on the amount of non-extracted resource, e.g., when the stock has an amenity value (D'Autume, Schubert, 2008). Then, if this value can be expressed in terms of utility, the problem can be reformulated by introducing the damage resulted from the resource extraction. In practice, this approach can be more precise, since the uncertainty in the extracted amount is essentially less than in the remaining stock. A review of studies with the modified HR can be found, e.g., in Gaudet (2007).

where  $v(t) := F_K - \delta$ ,<sup>11</sup>  $\tau(t)$ <sup>12</sup>  $= \tau[\mathbf{D}(t)] := D_5 + \partial D_4 / \partial(S_0 - S)$

$$+ \frac{1}{F_R} \left[ \frac{U_{D_2} \partial D_2 / \partial(S_0 - S)}{U_C} + F_{D_1} \frac{\partial D_1}{\partial(S_0 - S)} - \frac{\partial D_3}{\partial(S_0 - S)} \right], \quad (6)$$

and  $D_5 = 0$  if the economy is dynamically efficient.

**Proof** is in Appendix 1.

In this framework, dynamic efficiency is a relative notion. The planner's optimal path may be dynamically inefficient with respect to a first-best solution, for example, because the planner underestimates future production possibilities and considers the first-best path as infeasible (Section 3.1). The planner's optimal path may also appear inefficient when the planner does not account for the resource use externalities while estimating social progress. In the latter case, the planner may even consider the first-best path as inefficient due to the difference between the units of measure for utility in the planner's and in the first-best solutions (Section 3.2).

Deviation  $D_5$  may depend on  $\mathbf{D}$ , for example, on  $D_4$  when a planner does not take into account resource-augmenting investments ( $D_5 = -\partial D_4 / \partial(S_0 - S)$ , Section 3.1), or on  $D_1$  and  $D_2$  when the planner ignores the damages from the resource extraction (Section 3.2). In some cases, however,  $D_5$  may not depend on  $\mathbf{D}$ , and instead, both  $D_5$  and  $\mathbf{D}$  may be determined by the same phenomena, for example, imperfect institution (Section 3.3).

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<sup>11</sup> $v(t)$  is the market interest rate only with no distortion.

<sup>12</sup> $\tau(t)$  is the additive HR modifier or the influence of  $\mathbf{D}$ . This influence in equation (5) can be expressed in a multiplicative form:  $\dot{F}_R = v\eta[\mathbf{D}]F_R$ , where  $\eta[\mathbf{D}] := 1 + \tau[\mathbf{D}]/v$ . With no distortion,  $\tau = 0$ .

Genuine investment defined in Hamilton and Hartwick (2005) is

$$G(t) := \dot{K}(t) + \dot{S}(t)F_R(t). \quad (7)$$

This measure includes not only current investment into man-made capital but the value of the currently extracted resource estimated in marginal resource productivity, which, with no distortion, coincides with the market price. Hence,  $G$  indicates a combination of investment and resource policies.

Since utility may be distorted by  $D_2$ , the dependence of utility on consumption may be nonmonotonic; therefore, consumption cannot substitute utility as a measure of well-being (see, for example, Section 3.2). Hence, the proposition below establishes the link between  $G$  and  $\dot{U}$ , which includes the link between  $G$  and  $\dot{C}$  as a special case.

**Proposition 1.** *Current utility change is*

$$\dot{U} = (v - \dot{G}/G)GU_C + \Psi, \quad (8)$$

where  $\Psi := \dot{S}F_R U_C D_5$  is the influence of dynamic inefficiency.

**Proof** is in Appendix 2.

In a particular case, when the economy is dynamically efficient ( $D_5 = \Psi = 0$ ), formula (8) can be obtained from the results of Dixit et al. (1980) and Hamilton and Withagen (2007), expressed in present prices (see the proof). With no distortion, formula (8) coincides with the result of Hamilton and Hartwick (2005). In the general case, equation (8) shows that investment (7) can indeed determine  $\dot{U}$  if  $\Psi$  is relatively small. However,  $\dot{U}$  can be also completely determined by  $\Psi$  when the term  $(v - \dot{G}/G)GU_C$  is close to zero.

Of course, sharp changes in  $G$  can determine an *instant* sign of  $\dot{U}$  despite the large values of  $\Psi$ . Formula (8) shows that if there is a  $t = \bar{t}$ , such that  $\Psi(\bar{t})$  has a large positive (negative) value,  $\dot{U}(\bar{t})$  can be negative (positive) if  $G(\bar{t})$  is negative (positive) and  $\dot{G}(\bar{t})/G(\bar{t})$  has a large positive (negative) value. However, these cases are not relevant to sustainability due to the boundedness of investments, whereas distortions in general are less restricted. The boundedness of investment implies that the larger is  $\Psi$ , the shorter is the period of time when these cases are possible. Therefore, neglecting the short-run oscillations, it can be assumed, for determinateness, that  $|\dot{G}/G| < v < \infty$ ,<sup>13</sup> where  $v > 0$ , and the current investment  $\dot{K}$  is bounded by the current output  $Q$ . Then a *feasible investment* can be defined as follows.

**Definition 1.** *Investment  $\dot{K}(t) = w(t)Q(t)$  is feasible if  $w(t) \in (0, 1)$  and  $|\dot{G}/G| < v$  for any  $t \geq 0$ .*

Definition 1 results in the following Corollary.

**Corollary 1.** *Equation (8) implies that, for a feasible investment,*

$$\begin{aligned}
\text{(I)} \quad \dot{U} &\geq 0 \text{ iff} \\
\Psi &\geq -\left(v - \dot{G}/G\right) GU_C, \text{ or } G \geq -\dot{S}F_R D_5 / \left(v - \dot{G}/G\right), \text{ or} \quad (9) \\
D_5 &\begin{cases} \leq -\left(v - \dot{G}/G\right) \left(\dot{K} / \left(\dot{S}F_R\right) + 1\right), \text{ when } \dot{S} < 0; \\ \geq -\left(v - \dot{G}/G\right) \left(\dot{K} / \left(\dot{S}F_R\right) + 1\right), \text{ when } \dot{S} > 0; \end{cases} \quad (10)
\end{aligned}$$

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<sup>13</sup>The analysis can be easily complemented by the case with  $|\dot{G}/G| \geq v$ .

(II) *a feasible investment policy can change the sign of  $\dot{U}$  iff*

$$-\left(v - \frac{\dot{G}}{G}\right) < \left\{ D_5 \left\{ \begin{array}{l} < -\left(v - \frac{\dot{G}}{G}\right) \left(\frac{Q}{\dot{S}F_R} + 1\right), \text{ when } \dot{S} < 0; \\ < -\left(v - \frac{\dot{G}}{G}\right), \text{ when } \dot{S} > 0. \end{array} \right. \right. \quad (11)$$

The following examples show that the impact of dynamic inefficiency on the efficacy of investment depends on the level of output and the share of the resource rent in output. Assume that  $D_4 = 0$  ( $\dot{S} < 0$ ),  $v = 0.06$  and  $\dot{G}/G = 0.03$  at  $\bar{t} \geq 0$ .

**(a) Large resource-poor economy.** Let  $Q(\bar{t}) = 101$  and  $\dot{S}(\bar{t})F_R(\bar{t}) = -1$ . Then (Corollary 1) an investment policy can change the sign of  $\dot{U}(\bar{t})$  iff

$$-0.03 < D_5 < 3.$$

**(b) Small resource-rich economy.** In the case with  $Q(\bar{t}) = 11$  and  $\dot{S}(\bar{t})F_R(\bar{t}) = -10$ , an investment policy can affect the sign of  $\dot{U}(\bar{t})$  iff

$$-0.03 < D_5 < 0.003.$$

It is intuitive that a large economy has more opportunities in investment than a small one, and so the range for  $D_5$ , in which investment is able to affect the sign of utility change, is larger in case (a) than in case (b). Another difference between these two cases is that investment in a large resource-poor economy can change the sign of  $\dot{U}$  mostly when  $D_5$  affects  $\dot{U}$  negatively (positive  $D_5$  reduces  $\dot{U}$  when  $\dot{S} < 0$ ). In this example, the range of  $D_5 > 0$ , in which investment is able to compensate for the damage from the inefficiency,

is 100 times larger than the range of  $D_5 < 0$ , which positive effect on  $\dot{U}$  can be annihilated by negative  $G$ . This asymmetry is inverted in a small resource-rich economy.

Assume that  $\dot{S} < 0$ .<sup>14</sup> Then boundedness of investments implies that the current states of economies along the planner's paths belong to the one of the four following types determined by different roles of resource (institutional) and investment policies in the current change of utility depending on the level of inefficiency  $D_5$ .

(A)  $D_5 \geq -\left(v - \dot{G}/G\right) \left[Q/\left(\dot{S}F_R\right) + 1\right]$  : utility declines regardless of investment; non-negative values of  $\dot{U}$  can be obtained only by reduction of the inefficiency if it is still possible.<sup>15</sup>

(B)  $0 < D_5 < -\left(v - \dot{G}/G\right) \left[Q/\left(\dot{S}F_R\right) + 1\right]$  : utility growth can be achieved by investment policy alone; the optimal saving rate may be higher than under  $\Psi = 0$  in order to compensate not only for the shrinking natural capital but for the negative effect of inefficiency (for example, van der Ploeg 2011). Without a policy reducing  $D_5$ , the level of utility may be lower than under  $\Psi = 0$ .

(C)  $-\left(v - \dot{G}/G\right) < D_5 < 0$  : utility growth can be achieved by investment policy alone; the optimal saving rate may be lower than under  $\Psi = 0$  due to the positive effect from  $D_5$ ; decline in utility is still possible when

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<sup>14</sup>The inequalities below are inverted when  $\dot{S} > 0$ . An example that allows for the growing reserve stock is provided in Section 3.1. The case with  $\dot{S} > 0$  may lead to some interesting implications that require a separate study.

<sup>15</sup>Possibility of reduction of inefficiency depends on the state of the economy with respect to tipping points. This problem is not considered in the present paper.

$$G < -\dot{S}F_R D_5 / \left( v - \dot{G}/G \right) < 0.$$

(D)  $D_5 < -\left( v - \dot{G}/G \right)$  : utility grows regardless of investments; investment policy is important as a determinant of the level of utility along the growing path.

Types C and D may correspond to an economy where the planner underestimates future production possibilities (Section 3.1).

Condition (9) shows that, for  $\Psi < 0$ , the minimum investment  $G$  that provides non-declining utility can be essentially higher than zero. The next section shows that the value of  $G$  guaranteeing  $\dot{U} \geq 0$  may not exist.

### 3. Dynamic inefficiency and sustainability: Examples

In the examples below,  $D_5^*$  denotes a deviation of the ratio  $\dot{F}_R/F_R$  along the planner's optimal path from a first-best path and  $\tilde{D}_5$  — a deviation of the first-best path from the planner's optimal path ( $D_5^* = -\tilde{D}_5$ ). For succinctness, the planner's optimal paths and the models that do not take into account some of the distortions are called below “undistorted.”

#### 3.1. Resource-augmenting technical change

Assume that in a real economy,  $D_4$  is the only distortion, namely,  $\dot{S} = -R + S\phi(L_R/L)$ ,<sup>16</sup> where  $L_R/L$  is the share of the resource-augmenting research sector and  $\phi(\cdot) \geq 0$  is the rate of growth of the resource stock due to research (Takayama, 1980). According to (6), this problem yields condition (5) with  $\tau = -\phi$ .

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<sup>16</sup>Note that when  $\phi > R/S$ , the distortion  $D_4$  results in  $\dot{S} > 0$ .



Assume also that a social planner does not use the information about  $\phi$  and works with the undistorted model. Then, from the planner's point of view, the dynamics of the real economy without government intervention is non-optimal with  $\tilde{D}_5 = -\phi$ ,<sup>17</sup> which corresponds to the type C or D (depending on the behavior of  $\phi$ ) with growing consumption when  $G > \dot{S}F_R\phi / (v - \dot{G}/G)$  (the investment  $G$  may be negative here when  $\dot{S} < 0$ ). In fact, the reduced "optimal" planner's paths of resource extraction and consumption are dynamically inefficient with  $D_5^* = -\tilde{D}_5 = -\partial D_4 / \partial (S_0 - S) = \phi$ , and if  $D_4$  and a correspondent increment in consumption are taken into account, then  $D_5^* \equiv 0$  and consumption grows only when  $G > 0$ . Hence, in this example, the discrepancy between theory and real life results in a sustainable but inefficient path. Dynamic inefficiency can be reduced only by adjustment of the resource policy when the planner updates the information about reserve estimates. This adjustment can result in sustainable and asymptotically efficient economy.

### 3.2. Irreversible damages to utility and production

Assume that a social utility and production in a real economy are negatively affected by the damage  $D = D_1 = D_2$  caused by a stock externality<sup>18</sup> ( $D_{(S_0-S)} > 0, U_D < 0, F_D < 0$ ). If a planner disregards the damage, then, according to Lemma 1, the planner's paths are dynamically inefficient with  $D_5^* = -(F_D + U_D/U_C)D_{(S_0-S)}/F_R > 0$ .<sup>19</sup> The planner's problem reduces in

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<sup>17</sup>Utility is not distorted here ( $U_{D_2} = 0$ ); hence, formula (8) becomes  $\dot{C} = (v - \dot{G}/G)G + \dot{S}F_R D_5$ , since  $\dot{U} = U_C \dot{C} + U_{D_2} \dot{D}_2$ .

<sup>18</sup>E.g.,  $D$  may result from irreversible climate change (Stollery, 1998).

<sup>19</sup>As usual,  $U_C > 0$ .

this case to the one of Solow (1974) - Hartwick (1977), where, under the maximin criterion, the path of extraction starts from a higher level than in the efficient case,<sup>20</sup> and the economy follows a constant-consumption path (due to  $G = 0$ ) with a higher level than the initial level of the efficient path. Since for the planner,  $U_D = F_D = 0$ , formula (8) becomes  $\dot{C} = \left(v - \dot{G}/G\right) G$ .

In reality, however, the change in well-being is<sup>21</sup>

$$\dot{U} = \left(v - \dot{G}/G\right) G U_C + (F_D U_C + U_D) \dot{D}, \quad (12)$$

which is negative under the undistorted paths of investment and extraction ( $G = 0$ ) since  $(F_D U_C + U_D) \dot{D} < 0$ . Denote  $\Gamma = \min \dot{G}/G$  for all feasible  $G$ . Then, according to Corollary 1, investment  $G$  that provides  $\dot{U} > 0$  does not exist when the damage is large, namely, when

$$(F_D U_C + U_D) \dot{D} < -(v - \Gamma) \left(Q + \dot{S} F_R\right) < 0.$$

Hence, the undistorted policies result in inefficiency and unsustainability of this economy.

The growth of utility can be achieved in this case only when the planner recognizes the damage and reconsiders the measure of progress in the society. This done, the planner, situated in the Solow-Hartwick case, can obtain a sustainable and first-best optimal path by changing the resource policy alone,

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<sup>20</sup>When damage affects only production, Stollery (1998, p. 735) showed that the optimal extraction starts from a lower initial level and declines slower than in the case with no damage. The same result for damage in utility was obtained in Bazhanov (2011a, formula (33), Fig. 4).

<sup>21</sup>Formula (12) can be obtained from formula (8) using the expression for  $D_5^*$  and the fact that  $\dot{D} = -\dot{S} D_{(S_0-S)}$ .

namely, by reducing extraction, while the investment rule remains the same.<sup>22</sup>

### 3.3. Insecure property rights

Following Arrow et al. (2003, p. 664), assume that the owner  $i$  ( $i = 1 \dots N; N \geq 2$ ) extracts a liquid resource from the pool with the stock  $S_i$ . All  $N$  owners are identical, non-cooperative, and the pools are separated by porous barriers. The resource diffuses from larger pools to smaller ones with the same rate  $\lambda > 0$ .<sup>23</sup> Then the depletion equations are

$$\dot{S}_i = \lambda \sum_{j \neq i} (S_j - S_i) - R_i, i = 1 \dots N,$$

where  $R_i = R_i(t)$  is the rate of extraction of the owner  $i$  at the moment  $t$ . The necessary conditions for PV-maximization of the each owner's utility yield equation (5) with  $\tau \equiv D_5 = (N - 1)\lambda > 0$  (socially efficient paths require  $N = 1$ ). This inefficiency results in the distorted path of extraction

$$R_D = \sum_{i=1}^N R_i = [(\rho + D_5) / \eta] S_0 e^{-(\rho + D_5)t/\eta}$$

with the higher initial rate  $R_D(0)$  and faster decline  $\dot{R}_D$  than for the efficient path

$$R = [\rho / \eta] S_0 e^{-\rho t / \eta}.$$

In these formulas,  $\rho > 0$  is the social discount rate, and  $\eta > 1$  — the elasticity of marginal utility. Hence, the distorted equation for the whole reserve is  $\dot{S} = -R + D_4$ , where  $D_4 = R - R_D$ . In this example,  $D_4$  does not depend directly on the extracted resource  $S_0 - S$ , and deviation  $D_5$  does

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<sup>22</sup>Stollery (1998) showed that the Hartwick rule ( $G = 0$ ) is still optimal in this economy.

<sup>23</sup>No barriers corresponds to  $\lambda \rightarrow \infty$ .

not depend on  $D_4$ ; both these distortions result from imperfect institutions, expressed in  $N \geq 2$  and  $\lambda > 0$ .<sup>24</sup>

Let  $v = 0.06$ , and  $\dot{G}/G = -0.04$ . Since  $\dot{S} < 0$  and  $D_5 \geq 0$  for any  $t \geq 0$ , a feasible investment policy can change the sign of  $\dot{U}$  iff (Corollary 1)

$$(N - 1)\lambda < -0.1 \left[ Q / (\dot{S}F_R) + 1 \right] > 0.$$

When this condition is not satisfied, only institutional changes and resource policies can prevent decline in utility. An investment provides not declining utility here iff

$$G \geq -10(N - 1)\lambda \dot{S}F_R > 0 \quad \text{or} \quad w \geq -[10(N - 1)\lambda + 1] (\dot{S}F_R/Q),$$

which is very restrictive for  $N > 1$ .

It is illustrative to consider two cases.

(a) *Large resource-poor economy* ( $Q = 101$ ;  $\dot{S}F_R = -1$ ). In this case,  $\dot{K}$  can change the sign of  $\dot{U}$  iff  $(N - 1)\lambda < 10$ , which means, for example, that, for  $\lambda = 1$ , utility declines for any investment (type A) if  $N \geq 11$ . Let

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<sup>24</sup>Formally, the link  $D_5(D_4)$  is  $D_5 = -\rho - (\eta/t) W [D_4 t / S_0 - (\rho t / \eta) e^{-(\rho t / \eta)}]$ , where  $W[\cdot]$  is the Lambert W function. Numerically, using computational software (e.g., Maple), this formula gives  $D_5 = (N - 1)\lambda$  for any  $t$  when  $\arg\{W[\cdot]\} > -1/e$ . Also, formally,  $D_4$  changes with  $S_0 - S(t)$ , since both are changing in time. However, it can be shown that  $D_4$  cannot be represented as a function of  $S_0 - S$  only. Namely, the assumption  $D_4 = D_4(S_0 - S)$ , given  $\tau$ , the expression  $D_4 = R - R_D$ , and using Lemma 1, results in  $D_4 = (N - 1)\lambda(S_0 - S) + D_4(0)$ , since  $D_1 = D_2 = D_3 = 0$ . Here,  $D_4(0) = -(N - 1)\lambda S_0 / \eta$ . However, since the reserve  $S_0$  is fixed,  $D_4$  must result only in intertemporal redistribution of the resource, namely, the condition  $\int_0^\infty D_4 dt = 0$  must hold, which is not true for  $D_4$  derived in this way.

$N = 5$ . Then the saving rate, compensating for the shrinking resource and inefficiency, should be no less than  $w_{\min} = \frac{41}{101}$  (or  $G/Q \geq \frac{40}{101}$ ), whereas with no distortion ( $N = 1$ ), utility grows for any  $w > \frac{1}{101}$  (or  $G > 0$ ).

(b) *Small resource-rich economy* ( $Q = 11$ ;  $\dot{S}F_R = -10$ ).  $\dot{K}$  can change the sign of  $\dot{U}$  iff  $(N - 1)\lambda < 0.01$ ; for example, for  $\lambda \geq 0.01$  and  $N \geq 2$ , utility declines regardless of any feasible investment. Let  $\lambda = 0.009$  and  $N = 2$ . Then not declining utility is possible when almost all output is being invested, namely,  $w \geq \frac{10.9}{11}$ , although, for this resource-dependent economy, even with no distortion ( $N = 1$ ), the saving rate yielding at least constant utility must be very high, namely,  $w_{\min} = -\dot{S}F_R/Q \approx 0.91$ .

Hence, the use of the policies for the undistorted model in this example also results in inefficiency and unsustainability; moreover, a feasible investment compensating for the inefficiency and providing non-declining current utility may not exist.

#### 4. Indicator GI and sustainability evaluation

This section briefly discusses some questions that arise in the use of various forms of indicator GI in sustainability evaluation.

(a) *Aggregate consumption*. Proposition 1 can be generalized in a straightforward way for  $n$  resources. Then formula (7) becomes

$$G(t) := \dot{K}(t) + \sum_{i=1}^n \dot{S}_i(t)F_{R_i}(t), \quad (13)$$

and, applying the same approach for the proof, the combined influence of all the inefficiencies on utility is

$$\Psi := U_C \sum_{i=1}^n \dot{S}_i F_{R_i} D_{5i}. \quad (14)$$

Equations (8) and (14) show that when utility depends on the aggregate consumption, the combined effect of all  $D_{5i}$  on  $\dot{U}$  may be positive despite the unsustainable extraction of some resources. The problem originates from the assumption that the components of consumption are substitutes.<sup>25</sup> This assumption implies a specific aggregation of all  $D_{5i}$ , and this aggregation may not adequately reflect the impact of each source of inefficiency on  $\dot{U}$ . For example, if two resources are not substitutes in production (oil and copper), then, according to formula (14), a common pool situation for one resource can be compensated by the resource-augmenting investment for another resource. An indicator using this aggregation will show that the current utility should grow, despite the known problems in the future.

(b) *Disaggregate consumption.* A utility function that depends on a number of factors<sup>26</sup> is supposed to be more accurate for sustainability evaluation than a function that depends on aggregate consumption, but only if the specification of this function adequately reflects the preferences of a representative agent. The problems with the use of this function result, first, from complexities in defining empirically well-grounded functional forms of dependence of  $U$  on each factor, and, secondly, from increasing technical work in theoretical estimations. For example, the proof of Proposition 1 would require splitting the balance equation on “industries” that correspond to the consumption

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<sup>25</sup>This assumption is consistent with measuring social progress in terms of GDP – all components of consumption enter GDP in the same money units. In fact, some components of consumption, e.g. transport, food, and health, are not substitutes.

<sup>26</sup>For this kind of function,  $\dot{U} = \sum_i U_{C_i} \dot{C}_i + \sum_j U_{D_{2j}} \dot{D}_{2j}$ , where each  $C_i$  affects utility positively and  $D_{2j}$  – negatively.

components.

(c) *Indicators GI based on CUC and on change in intergenerational welfare.* Indicator GI based on CUC may not show the change in the ability of an economy to maintain non-declining utility during a long period of time. For example, in the DHSS economy ( $Q = F(K, R) = K^\alpha R^\beta$ ), indicator (7) expressed as a share of GDP takes the form  $G/Q = w - \beta$ , where  $w$  is the rate of investment ( $\dot{K} = wQ$ ). Assume, for simplicity, that utility monotonically depends on consumption (no damages). An economic program with non-declining consumption exists in this economy at  $t = t_0$  if and only if<sup>27</sup>

$$G_\infty/Q|_{t=t_0} := w - KR\beta/[S_0Q(\alpha - \beta)]|_{t=t_0} \geq 0. \quad (15)$$

Hence, this economy is *potentially sustainable* at  $t = t_0$  (a program with non-declining utility exists) if and only if  $R(t_0)/S_0 \leq wQ(\alpha - \beta)/[\beta K]|_{t=t_0}$ , which means that potential sustainability can be achieved by a change in resource policy only, regardless of the value of  $G/Q|_{t=t_0}$  and the sign of  $\dot{C}(t_0)$ . The indicators  $G$  and  $G_\infty$  coincide if and only if the state of the economy satisfies a “perfection” condition ( $R/S_0 = Q(\alpha - \beta)/K$ ) with respect to a constant-consumption criterion (Bazhanov 2010).

On the other hand, the indicator  $G_\infty$  does not reflect CUC and current level of consumption (utility). As a result, a positive value of this indicator may “approve” a sharp decline in the current rate of resource extraction,

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<sup>27</sup>Derivation of the expression for  $G_\infty$ , which is called the *level of potential sustainability*, is provided in Bazhanov (2011b). This expression coincides with the indicator GI where the resource price shows the change in the maximum level of consumption that can be maintained forever.

which may lead to a drop in the current level of utility below a subsistence minimum violating intergenerational justice. Of course, unsustainability of an economy ( $G_\infty < 0$ ) requires changes in investment and resource policies to increase the value of  $G_\infty$ , but this requirement raises the questions: 1) what is the maximum value of  $G_\infty$  for an optimal economic growth, and 2) what is the path of acceptable sacrifices in current utility for future sustainable development?

## 5. Concluding remarks

This study assumed that a social planner of a resource-based economy constructed optimal paths using a model that is not sufficiently adequate to the problem, for example, due to imperfection in knowledge or in institutions. In order to imitate this discrepancy between theory and real life, the study has extended the result of Hamilton and Hartwick (2005) regarding the role of genuine investment (GI) in current utility change (CUC) by assuming that 1) there are distortions that affect utility, production, balance equation, and the dynamics of the resource stock; and 2) the planner does not take into account some of these distortions. As a result, the planner's paths are only second-best optimal and dynamically inefficient. Proposition 1 shows the link between GI and CUC depending on the influence of this inefficiency.

Proposition 1 and Corollary 1 have shown that CUC can be determined by GI, measured in the marginal resource productivity, only when the influence of the disregarded distortions is close to zero. These results entail a classification of the status of a resource-based economy by the importance of investment or resource policies or both for CUC. The study has shown



that the distortions asymmetrically affect the ability of investment to control CUC, and this asymmetry is mutually inverted for capital-rich-resource-poor and capital-poor-resource-rich economies.

The examples of neglected distortions demonstrated that a dynamically inefficient economy may be sustainable, when the planner underestimates future production possibilities, or unsustainable, when the planner ignores institutional imperfections or damages from economic activities. The examples also have shown that in the presence of inefficiencies, a feasible level of GI that provides a positive CUC may not exist.

Since, by the definition, an economy is unsustainable if utility declines at any moment of time (including the current moment), the results of this study imply that for sustainability of real economies, 1) institutional and resource policies may be more important than investment policies when the level of inefficiency is high; 2) it is preferable that a resource policy is more conservative than is prescribed by a theory. In the former conclusion, investment policy, of course, is still important as a determinant of the level of utility along a growing or declining path and as a determinant of growth when the level of inefficiency is low. In the latter one, an overly conservative resource policy may result in dynamic inefficiency, but with updates in knowledge, the policy can be corrected, and the economy can be asymptotically efficient.

Besides regular consequences caused by deviation of real life from theory, an indicator based on CUC may not reflect sustainability of an economy, because, by construction, it does not show the change in the ability of the economy to maintain non-declining utility during a long period of time. According to Arrow et al. (2003), in order to evaluate current sustainability

change, the accounting prices should be used for measuring GI. However, an indicator GI based on these prices may not reflect CUC, which may result in an increase in the long-term welfare at the cost of an unacceptable decline in current utility, violating the principles of sustainability and intergenerational justice. The properties of different forms of indicator GI imply that these indicators can complement each other in sustainability evaluation, or another indicator (or algorithm of sustainability evaluation) that is consistent with a criterion of intergenerational justice at any moment of time can be constructed.

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## 7. Appendix 1. Proof of Lemma 1

Since optimal paths are always efficient, a necessary condition of dynamic efficiency for economy (1)-(4) can be obtained from optimality conditions, for example, in the problem of PV-maximization<sup>28</sup> of  $\int_0^\infty U(C, D_2)\pi(t)dt$  with a discount factor  $\pi(t)$ . The Hamiltonian of this problem is  $H = U(C, D_2)\pi(t) + \mu_K(F - C - \delta K - D_3) + \mu_S(D_4 - R)$ , and the Pontryagin-type necessary

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<sup>28</sup>The maximin, formulated as  $\max_{r,c} \int_0^\infty \bar{U} \delta e^{-\delta t} dt \equiv \bar{U} = \text{const}(r, c)$  with the additional constraint  $U(C, D_2) = \bar{U}$ , yields the same result.

conditions are

$$H_C = U_C \pi(t) - \mu_K = 0, \quad (16)$$

$$H_R = \mu_K F_R - \mu_S = 0, \quad (17)$$

$$\dot{\mu}_K = -\frac{\partial H}{\partial K} = -\mu_K (F_K - \delta), \quad (18)$$

$$\begin{aligned} \dot{\mu}_S = & -\frac{\partial H}{\partial S} = -\pi(t) U_{D_2} \frac{\partial D_2}{\partial (S_0 - S)} \frac{\partial (S_0 - S)}{\partial S} - \\ & \mu_K \left( F_{D_1} \frac{\partial D_1}{\partial (S_0 - S)} \frac{\partial (S_0 - S)}{\partial S} - \frac{\partial D_3}{\partial (S_0 - S)} \frac{\partial (S_0 - S)}{\partial S} \right) - \\ & \mu_S \frac{\partial D_4}{\partial (S_0 - S)} \frac{\partial (S_0 - S)}{\partial S}. \end{aligned} \quad (19)$$

Equation (19) with  $\mu_K$  from (16) becomes

$$\begin{aligned} \dot{\mu}_S = & \pi(t) U_{D_2} D_{2(s_0-s)} + \\ & U_C \pi(t) \left( F_{D_1} D_{1(s_0-s)} - D_{3(s_0-s)} \right) + \mu_S D_{4(s_0-s)}. \end{aligned} \quad (20)$$

The time derivative of equation (17) is  $\dot{\mu}_S = \dot{\mu}_K F_R + \mu_K \dot{F}_R$ , which, combined with (20), results in

$$\begin{aligned} \dot{\mu}_K F_R + \mu_K \dot{F}_R = & \pi(t) \left[ U_{D_2} D_{2(s_0-s)} + U_C \left( F_{D_1} D_{1(s_0-s)} - D_{3(s_0-s)} \right) \right] + \\ & \mu_S D_{4(s_0-s)}. \end{aligned}$$

The last equation after dividing through by  $F_R$  and substitutions for  $\dot{\mu}_K$  (from (18)) and  $\mu_S$  (from (17)) becomes

$$\begin{aligned} -\mu_K (F_K - \delta) + \mu_K \frac{\dot{F}_R}{F_R} = & \frac{\pi(t)}{F_R} \left[ U_{D_2} D_{2(s_0-s)} + U_C \left( F_{D_1} D_{1(s_0-s)} - D_{3(s_0-s)} \right) \right] + \\ & \mu_K D_{4(s_0-s)}, \end{aligned}$$

which, divided through by  $\mu_K$  with substitution for  $\mu_K$  from (16), yields  $\dot{F}_R/F_R = F_K - \delta + \tau(\mathbf{D})$ , where  $\tau(\mathbf{D})$  is defined by formula (6) ■

## 8. Appendix 2. Proof of Proposition 1

The proof follows the approach of Hamilton and Hartwick (2005, Proposition 1), which was first applied in Hartwick (1977). The differences are that the current proof uses: 1) utility as a measure of well-being (due to distortion  $D_2$ ); 2) a modified HR to substitute for  $\dot{F}_R$  instead of the standard HR. Namely, equations (1), (3), and (5) give

$$\begin{aligned}\dot{C} &= \dot{K}F_K + \dot{R}F_R + \dot{D}_1F_{D_1} - \delta\dot{K} - \ddot{K} - \dot{D}_3 \\ &= \dot{K}F_K + \dot{R}F_R + \dot{D}_1F_{D_1} - \delta\dot{K} - \ddot{K} - \dot{D}_3 + R\dot{F}_R - R\dot{F}_R \\ &= (F_K - \delta)\dot{K} - (F_K - \delta + \tau)F_R R - \left[ \ddot{K} - d(RF_R)/dt \right] + \dot{D}_1F_{D_1} - \dot{D}_3.\end{aligned}$$

From (4),  $R = -\dot{S} + D_4$ . Then

$$\begin{aligned}\dot{C} &= (F_K - \delta)\dot{K} - (F_K - \delta + \tau)F_R(-\dot{S} + D_4) \\ &\quad - \left[ \ddot{K} - d\left\{(-\dot{S} + D_4)F_R\right\}/dt \right] + \dot{D}_1F_{D_1} - \dot{D}_3 \\ &= (F_K - \delta)\dot{K} + (F_K - \delta)F_R\dot{S} - (F_K - \delta)F_RD_4 - \tau F_R R \\ &\quad - \left[ \ddot{K} + d\left\{\dot{S}F_R\right\}/dt \right] + d\{D_4F_R\}/dt + \dot{D}_1F_{D_1} - \dot{D}_3\end{aligned}$$

or, by the definition of  $G$  given in (7),

$$\dot{C} = vG - \dot{G} - vF_RD_4 + \dot{D}_4F_R + D_4\dot{F}_R + \dot{D}_1F_{D_1} - \dot{D}_3 - \tau F_R R. \quad (21)$$

Substitution for  $\dot{F}_R$  from (5) yields

$$\dot{C} = vG - \dot{G} + \tau F_RD_4 + \dot{D}_4F_R + \dot{D}_1F_{D_1} - \dot{D}_3 - \tau F_R R,$$

which, using (4), becomes

$$\dot{C} = vG - \dot{G} + \dot{D}_4 F_R + \dot{D}_1 F_{D_1} - \dot{D}_3 + \tau F_R \dot{S}. \quad (22)$$

Assumptions 1 and 2 imply that  $\dot{D}_i = -\dot{S} \partial D_i / \partial (S_0 - S)$ . Then, using Lemma 1,

$$\tau F_R \dot{S} = D_5 F_R \dot{S} - \dot{D}_4 F_R - \dot{D}_1 F_{D_1} + \dot{D}_3 - U_{D_2} \dot{D}_2 / U_C.$$

Substitution of this expression into (22) results in

$$\dot{C} = vG - \dot{G} - U_{D_2} \dot{D}_2 / U_C + D_5 F_R \dot{S},$$

which, after substitution into  $\dot{U} = U_C \dot{C} + U_{D_2} \dot{D}_2$ , yields formula (8) of the proposition.

When economy (1)-(4) is dynamically efficient ( $D_5 = 0$ ), formula (8) can be obtained from the result of Dixit et al. (1980, Theorem 1) or from a generalization of this result in Hamilton and Withagen (2007). Namely, in terms of the present value prices of utility, capital, and the resource, defined in Lemma 1 as  $\pi(t)$ ,  $\mu_K$ , and  $\mu_S$ , these results claim that

$$\pi(t) \dot{U} = -\frac{d}{dt} \left( \mu_K \dot{K} + \mu_S \dot{S} \right),$$

which, using formulas (16)-(18), can be rewritten as follows:

$$\begin{aligned} \pi(t) \dot{U} &= -\frac{d}{dt} \left[ \mu_K \left( \dot{K} + \frac{\mu_S}{\mu_K} \dot{S} \right) \right] = -\frac{d}{dt} \left[ \mu_K \left( \dot{K} + F_R \dot{S} \right) \right] \\ &= -\frac{d}{dt} [\mu_K G] = -\mu_K \left( \frac{\dot{\mu}_K}{\mu_K} G + \dot{G} \right) = \mu_K \left[ (F_K - \delta) G - \dot{G} \right]. \end{aligned}$$

Then, with the use of formula (16) and the notation  $v := F_K - \delta$ , it becomes

$$\dot{U} = \left( v - \dot{G}/G \right) G U_C,$$

which is formula (8) for  $D_5 = 0$  ■

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